panding Eq. (6) for low frequencies, in particular $w^* = w^{(0)} + ikw^{(1)}$, yields

$$W^{(0)} = w^{(0)} \quad (13)$$

$$W^{(1)} = w^{(1)} + \frac{M_\infty^2}{M_\infty^2 - 1} xw^{(0)} + \frac{1}{1 - M_\infty^2} \int_{-\infty}^{x} w^{(0)} d\xi \quad (14)$$

It remains to express the unsteady potential ϕ^* in terms of steady potentials $\Phi^{(0)}$ and $\Phi^{(1)}$. A first order in frequency expansion of Eq. (5), where ϕ^* is expanded as $\phi^* = \phi^{(0)} + ik\phi^{(1)}$, results in

$$\phi^{(0)} = \Phi^{(0)} \quad (15)$$

$$\phi^{(1)} = \Phi^{(1)} + \frac{M_\infty^2}{1 - M_\infty^2} x\Phi^{(0)} + \frac{1}{M_\infty^2 - 1} \int_{-\infty}^{x} \Phi^{(0)} d\xi \quad (16)$$

The desired reduction formula for the unsteady velocity potential is therefore

$$\phi^* = \Phi^{(0)} + ik \left(\Phi^{(1)} + \frac{M_\infty^2}{1 - M_\infty^2} x \, \Phi^{(0)} + \frac{1}{M_\infty^2 - 1} \times \int_{-\infty}^{x} \Phi^{(0)} d\xi \right) \quad (17)$$

The author[7] showed that a low-frequency expansion of ϕ^* of this type is valid for finite wings in subsonic flow and arbitrary wings in supersonic flow. The frequency is strongly restricted beyond the low-frequency assumption $k \ll 1$ by the Mach number. $\Phi^{(0)}$ and $\Phi^{(1)}$ are potentials of wings in steady flight with upwashes given by Eqs. (13) and (14). From Eq. (17) follows the unsteady pressure amplitude in terms of steady pressures as

$$\Delta c_p^* = \Delta C_p^{(0)} + ik \left(\Delta C_p^{(1)} + \frac{M_\infty^2}{1 - M_\infty^2} x \, \Delta C_p^{(0)} \right) \quad (18)$$

where $\Delta C_p^{(0)}$ and $\Delta C_p^{(1)}$ are zeroth and first-order terms of the modified pressure jump $\Delta C_p = \Phi_x$.

Miles' Reduction Formula

Equation (5) can be identified as the application of a well-known modification to the pressure or acceleration potential $\phi_x^* + ik\phi^*$. The same modification applied to the velocity potential ϕ^*, as done by Miles,[6] does not yield a complete reduction to steady flow. Miles' supersonic formula reads

$$\phi^*(x,y,z) = \phi_s(x,y,z;w^{(0)}) + ik \left\{ \phi_s(x,y,z;w^{(1)}) + \frac{M_\infty^2}{M_\infty^2 - 1} [\phi_s(x,y,z;xw^{(0)}) - x\phi_s(x,y,z;w^{(0)})] - \frac{1}{M_\infty^2 - 1} \int_{-\infty}^{x} [\phi_{qs}(\xi,y,z;w^{(0)}) - \phi_s(\xi,y,z;w^{(0)})] d\xi \right\} \quad (19)$$

where the unsteady upwash is already expanded to first order in frequency. ϕ_s is a steady potential. ϕ_{qs} is quasisteady, i.e., it is a solution of a steady boundary value problem in which the wake condition is replaced by $\phi_{qs}|_{z=0} = 0$. The notation $\phi_s(x,y,z;w^{(0)})$ indicates that ϕ_s is the potential of a wing with upwash $w^{(0)}$.

Comparing Eq. (19) with the new reduction formula (17) shows that the quasi-steady potential in Miles' formula can be replaced by a steady potential.

$$\int_{-\infty}^{x} \phi_{qs}(\xi,y,z;w^{(0)}) d\xi = \phi_s\left(x,y,z; \int_{-\infty}^{x} w^{(0)} d\xi\right) \quad (20)$$

The steady potential ϕ_s satisfies the boundary condition on the lifting surface

$$\phi_{s_z}|_{z=0} = \int_{-\infty}^{x} w^{(0)}(\xi,y) d\xi \quad (21)$$

Conclusions

The low-frequency problem in unsteady aerodynamics has been reduced completely to a pair of familiar steady wing problems. The reduction formula for the velocity potential of slowly oscillating wings at subsonic and supersonic speeds is given by Eq. (17). For known unsteady upwash, the upwashes of the equivalent steady wings are obtained from Eqs. (13) and (14). The reduction formula can be used to apply existing steady aerodynamic programs for planar or nonplanar configurations to the problem of calculating dynamic stability derivatives. For this first order in frequency approximate theory, the stability derivatives do not depend on the reduced frequency. Hence, they may be used to compute stability characteristics by the usual solution of an eigenvalue problem.

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Technical Comment

Erratum: "Sonic Boom Minimization Schemes"

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UNFORTUNATELY, the following transcription errors escaped the proofreading process.[1] Equation (2) should read

$$\dot{m}(x) = \rho_\infty U_\infty \int_0^x [1 + \tfrac{1}{2} M_\infty^2 C_{p0}(t)] \Delta A'(t) dt$$

Equation (3c) should read

$$\frac{\gamma}{\gamma - 1} \rho u A \left(\frac{1}{\rho} \frac{dP}{dx} - \frac{P}{\rho^2} \frac{d\rho}{dx} + \frac{\gamma - 1}{\gamma} u \frac{du}{dx} \right) = \dot{q}(x)$$

Reference

[1] Siegelman, D., "Sonic Boom Minimization Schemes," *Journal of Aircraft*, Vol. 7, No. 3, May–June 1970, pp. 280–281.